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Effects of viscous dissipation on the stability of a liquid film flowing down a heated inclined plane

D. A. GOUSSIS
Mechanical, Aerospace, and Nuclear Engineering Department, University of California, Los Angeles, CA 90024, U.S.A.

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Abstract—The effects of viscous dissipation on the linear stability of a liquid film flowing down a heated inclined plane are examined. It is shown that for the thermal mode of instability viscous dissipation has both stabilizing and destabilizing influences. The overall effect on the stability of the flow depends on the value of the Prandtl number.

1. INTRODUCTION

A LIQUID film flowing down an inclined heated plane is subject to a hydrodynamic mode of instability, which can occur in the isothermal case, and also to a thermal mode of instability, which can occur in the presence of a temperature gradient when the plane is horizontal. These two possible modes of instability were examined by Kelly and Goussis [1] on a linear basis and within the confines of the Boussinesq approximation. It was shown that heating has only a higher order effect on the hydrodynamic mode. As a result, the stability of the flow with respect to that mode is determined by the results of the homogeneous problem for which it has been shown that the instability assumes the form of transverse waves. Furthermore, it was shown that the free surface deflection has only a higher order effect on the thermal mode of instability. For this mode, the basic flow tends to stabilize the disturbances whose vector has a component in the streamwise direction. As a result, the instability assumes the form of stationary spanwise-periodic longitudinal rolls. When surface waves constitute the hydrodynamic mode, Kelly and Goussis [1] determined a transition angle of inclination β_{tr}, beyond which the hydrodynamic mode is dominant and below which the thermal mode is dominant. This angle is given by

$$\beta_{tr} = \tan^{-1}\left[\frac{5\alpha\Delta T Pr}{2Ra_{c,\beta=0}}\right]^{1/2} \quad (1)$$

where ΔT is the temperature difference across the film, $Ra_{c,\beta=0}$ denotes the critical Rayleigh number at zero angle of inclination. As equation (1) shows, β_{tr} is in general small except for very viscous liquids.

Here, the effects of viscous dissipation on the linear stability of the film flow will be examined. These effects will be regarded as a departure from the Boussinesq approximation while all the other higher order effects will be ignored. When $\beta > \beta_{tr}$, the velocity field for the more unstable disturbance is uncoupled from the temperature field [1]. As a result, inclusion of the viscous dissipation term in the energy equation will have no effect on the stability of the flow. Therefore, only the thermal mode of instability, which is the dominant mode when $\beta < \beta_{tr}$, needs to be considered.

In the absence of viscous dissipation the onset of the thermal mode of instability for the film flow problem is independent of the basic velocity profile and is governed by a set of equations similar to that governing the Rayleigh–Benard problem [1]. In the latter problem, the effects of viscous dissipation can be considered only on a non-linear basis since the term which expresses these effects in the energy equation involves the squares of disturbance quantities. The effects of viscous dissipation on the non-linear Rayleigh–Benard problem were considered by Turcotte *et al.* [2] along with the effects of an adiabatic temperature gradient. They showed that both effects are characterized by the same non-dimensional parameter, namely, the dissipation number so that, at least in the non-linear context, they have to be considered simultaneously. Their numerical calculations of finite amplitude convection show that in the infinite Prandtl number limit, increasing values of the dissipation number decrease the velocity and finally stabilize the layer. The effects of an adiabatic temperature gradient on the Rayleigh–Benard problem can be considered on a linear basis. Jeffreys [3] proved that the resulting problem is equivalent to one with a smaller basic temperature gradient which therefore yields a more stable flow. It can be shown that Jeffreys' treatment of the effects of an adiabatic temperature gradient is still applicable in the problem considered here and yields exactly the same results as in ref. [3]. Therefore, we can proceed with an examination of the effects of viscous dissipation only.

NOMENCLATURE

c_p	specific heat at constant pressure
D	d/dz
F	amplitude of disturbance velocity $\hat{u}$
g	gravitational acceleration
G	amplitude of disturbance velocity $\hat{v}$
h	heat transfer coefficient
H	amplitude of disturbance velocity $\hat{w}$
k	wavenumber
K	thermal conductivity
L	film layer depth
p	pressure
T	temperature
ΔT	reference temperature difference
t	time
u	velocity component in x-direction
v	velocity component in y-direction
V_r	reference velocity, $gl^2 \sin\beta/(2\nu)$
w	velocity component in z-direction
x, y, z	Cartesian coordinates.

Greek symbols

α	coefficient of thermal expansion
β	angle of inclination
θ	amplitude of disturbance temperature
κ	thermal diffusivity
μ	dynamic viscosity
ν	kinematic viscosity
ρ	density.

Dimensionless groups

Bi	Biot number, hL/K
Di	Dissipation number, $\alpha gL/c_p$
Pr	Prandtl number, ν/κ
Ra	Rayleigh number, $\alpha g\Delta T_0 L^3/\nu\kappa$
λ	dimensionless group, $k\,Ra^{1/2}$
Λ	dimensionless group, $\sin\beta/(2\alpha\Delta T_0)$.

Superscripts

$\bar{}$	non-dimensional steady state quantities
$\hat{}$	non-dimensional perturbation quantities.

Subscripts

c	critical values
y, z	partial derivative
w	value at the heated wall
∞	value at the ambient conditions.

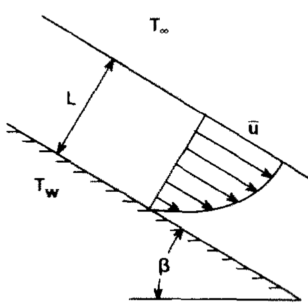

FIG. 1. Configuration of the flow.

Since a basic velocity exists for the film flow problem, such an analysis can be done on a linear basis. In Section 2 the linear perturbation equations are presented and corrections, which account for the effects of viscous dissipation, on the critical Rayleigh number and wavenumber are derived. The results are discussed in Section 3.

2. FORMULATION OF THE PROBLEM

The configuration of the flow examined here is depicted in Fig. 1. Considering the depth of the film L as the length scale and introducing the non-dimensional variables

$$u = V_r\bar{u}, \quad T = \Delta T\bar{T} + T_w, \quad \Delta T = T_w - T(0) \qquad \text{(2a–c)}$$

the governing equations and boundary conditions for the basic flow become

$$D^2\bar{u} = -2, \quad \bar{u}(1) = 0, \quad D\bar{u}(0) = 0 \qquad \text{(3a–c)}$$

$$D^2\bar{T} = -Di\,Ra\,\Lambda^2(D\bar{u})^2, \quad \bar{T}(1) = 0,$$
$$D\bar{T}(0) = Bi[\bar{T}(0) + R] \qquad \text{(4a–c)}$$

where $R = (T_w - T_\infty)/\Delta T$. The solutions to the above equations are

$$\bar{u} = 1 - z^2, \quad \bar{T} = (z-1) + Di\,Ra\,\Lambda^2(z - z^4)/3. \qquad \text{(5a,b)}$$

From the scalings, equations (2b) and (2c), and the definition of R we obtain

$$\Delta T = \Delta T_0\left[1 - Di\frac{Ra\,\Lambda^2}{3(1+Bi)}\right] \qquad (6)$$

where $\Delta T_0 = (T_w - T_\infty)Bi/(1 + Bi)$.

Assuming that, as is the $Di = 0$ case [1], the basic flow stabilizes all but the spanwise-periodic disturbances and that the disturbance at the marginal state is stationary, the governing non-dimensional stability equations are

$$\hat{v}_y + \hat{w}_z = 0 \tag{7a}$$

$$\frac{Ra\,\Lambda}{Pr}(1 + Di\,S)(\hat{w}D\bar{u}) = \nabla^2\hat{u} \tag{7b}$$

$$0 = \nabla^2\hat{v} - \hat{p}_y \tag{7c}$$

$$0 = \nabla^2\hat{w} - \hat{p}_z - Ra\,\hat{T} \tag{7d}$$

$$\hat{w}D\bar{T} = \nabla^2\hat{T} + 2\Lambda\,Di(D\bar{u})\hat{u}_z \tag{7e}$$

where $S = -Ra\,\Lambda^2/[3(1 + Bi)]$. These equations are supplemented by the non-slip and isothermal conditions at the wall

$$\hat{u} = \hat{v} = \hat{w} = \hat{T} = 0 \qquad \text{at } z = 1 \tag{7f}$$

and by the thermal, shear, and normal stress conditions at the free surface

$$\hat{T}_z = Bi\,\hat{T} \qquad \text{at } z = 0 \tag{7g}$$

$$\hat{u}_z = \hat{v}_z + \hat{w}_y = 0 \qquad \text{at } z = 0 \tag{7h}$$

$$\hat{w} = 0 \qquad \text{at } z = 0. \tag{7i}$$

In the derivation of the stability equations (7a)–(7i), apart from the scaling, equations (2a)–(2c), a diffusive scaling was used for the disturbance quantities [1]. By setting either Λ or Di equal to zero in equations (7a)–(7i), the Rayleigh–Benard problem is recovered in the form considered by Sparrow *et al.* [4]. Here the effects of viscous dissipation will be regarded as a small departure from the Boussinesq approximation. Hence, after representing the disturbance quantities in the usual way

$$(\hat{u}, \hat{v}, \hat{w}, \hat{T}) = [F(z), G(z), H(z), \theta(z)]\exp(\mathrm{i}\,ky) \tag{8}$$

an expansion of the variables and dependent parameters is made in the following manner

$$(\bar{T}, F, G, H, \theta, Ra) = \Sigma(Di)^j(\bar{T}_j, F_j, G_j, H_j, \theta_j, Ra_j). \tag{9}$$

Substituting equations (8) and (9) into equations (7a)–(7i), rescaling F and θ for convenience as

$$F_j = \tilde{F}_j(Ra_0\,\Lambda/Pr), \quad \theta_j = \tilde{\theta}_j/(k^2\,Ra)^{1/2} \tag{10}$$

and collecting terms of the same powers in Di, the following sets are obtained:

zero-order problem

$$\mathrm{i}\,kG_0 + DH_0 = 0 \tag{11a}$$

$$(D^2 - k^2)\tilde{F}_0 = (D\bar{u})H_0 \tag{11b}$$

$$(D^2 - k^2)^2H_0 + \lambda_0\tilde{\theta}_0 = 0 \tag{11c}$$

$$(D^2 - k^2)\tilde{\theta}_0 - \lambda_0(D\bar{T}_0)H_0 = 0 \tag{11d}$$

$$\tilde{F}_0(1) = H_0(1) = DH_0(1) = \tilde{\theta}_0(1) = 0 \tag{11e–i}$$

$$D\tilde{F}_0(0) = H_0(0) = D^2H_0(0) = D\tilde{\theta}_0 - Bi\,\tilde{\theta}_0(0) = 0, \tag{11j–m}$$

first-order problem

$$\mathrm{i}\,kG_1 + DH_1 = 0 \tag{12a}$$

$$(D^2 - k^2)^2H_1 + \lambda_0\tilde{\theta}_1 = r_1 \tag{12b}$$

$$(D^2 - k^2)\tilde{\theta}_1 - \lambda_0(D\bar{T}_0)H_1 = r_2 \tag{12c}$$

$$H_1(1) = DH_1(1) = \tilde{\theta}_1(1) = 0 \tag{12d–f}$$

$$H_1(0) = D^2H_1(0) = D\tilde{\theta}_1(0) - Bi\,\tilde{\theta}_1(0) = 0 \tag{12g–k}$$

where

$$r_1 = -\frac{1}{\lambda_0}[\lambda_1^2 + \lambda_0^2 S]\tilde{\theta}_0 \tag{12l}$$

$$r_2 = \lambda_0 H_0 D\bar{T}_1 - \lambda_0\left(\frac{2Ra_0\Lambda^2}{Pr}\right)(D\bar{u}D\tilde{F}_0) \tag{12m}$$

$$\lambda_j = (k^2\,Ra_j)^{1/2}. \tag{12n}$$

In order to find the correction Ra_1 to the neutral Ra_0 due to the viscous dissipation effects, a solvability condition must be applied to the first-order problem. For this purpose the solution to the adjoint homogeneous problem is required. Letting

$$L = \begin{bmatrix}(D^2 - k^2)^2 & 0 \\ 0 & (D^2 - k^2)\end{bmatrix}, \quad M = \begin{bmatrix}0 & \lambda_0 \\ -\lambda_0 & 0\end{bmatrix}, \quad X = \begin{bmatrix}H_0 \\ \tilde{\theta}_0\end{bmatrix}$$

the adjoint homogeneous problem is obtained through the following condition

$$\langle X_0^*(L + M)X_0 - X_0(L^* + M^*)X_0^*\rangle = 0 \tag{13}$$

where L^*, M^*, and X^* denote the adjoint operators and solution vector and $\langle\rangle$ denote integration over the film depth. After some manipulations it can be shown that

$$H_0^* = H_0 \text{ and } \tilde{\theta}_0^* = -\tilde{\theta}_0.$$

The solvability condition is obtained by multiplying the governing equations of the first-order problem by the adjoint solution and integrating over the film depth, i.e.

$$\langle X_0^*(L + M)X_1\rangle = \langle X_0^*R\rangle \tag{14}$$

where

$$X_1 = [H_1, \tilde{\theta}_1]^{\mathrm{T}} \text{ and } R = [r_1, r_2]^{\mathrm{T}}.$$

Integrating by parts and using the boundary conditions, the following expression for Ra_1 is obtained

$$\frac{Ra_1 + Ra_0\,S}{Ra_0} = \Lambda^2\left[\frac{N_1}{Pr} + N_2\right] = E \tag{15}$$

where

$$N_1 = 2Ra_0 I_2/I_1, \qquad N_2 = -Ra_0 I_3/(3I_1) \qquad (16a,b)$$

$$I_1 = \langle H_0 \tilde{\theta}_0 \rangle, \qquad I_2 = \langle (D\bar{u})(D\tilde{F}_0)\tilde{\theta}_0 \rangle \qquad (17a,b)$$

$$I_3 = \langle (D\bar{T}_1) H_0 \tilde{\theta}_0 \rangle, \qquad \bar{T}_1 = z - z^4. \quad (17c,d)$$

Up to this point the Rayleigh number was based on the temperature difference ΔT_0. However, the actual temperature drop across the film is ΔT and is given by equation (6). In order to consider ΔT instead of ΔT_0 in the corrected Rayleigh number, we define

$$Ra^+ = Ra\frac{\Delta T}{\Delta T_0} \simeq Ra_0 + Di[Ra_1 + Ra_0 S]. \quad (18)$$

Equations (15) and (18) yield

$$Ra^+ = Ra_0(1 + Di\,E) \qquad (19)$$

where E was defined in equation (15). For fixed values of Λ, Bi, Di, and Pr, Ra^+ will be a function of the wavenumber. We are interested in finding the minimum value of Ra^+, say Ra_c^+, which occurs at $k = k_c$. Expanding k in powers of Di

$$k = k_0 + Di\,k_1 \qquad (20)$$

equation (19) yields

$$\left.\frac{dRa^+}{dk}\right|_{k=k_c} = \left[\left[\frac{dRa_0}{dk} + Di\,k_1\frac{d^2Ra_0}{dk^2}\right](1 + Di\,E) + Di\,Ra_0\frac{dE}{dk}\right]_{k=k_{c0}}. \qquad (21)$$

However

$$\frac{dRa^+}{dk} = 0 \text{ at } k = k_c \quad \text{and} \quad \frac{dRa_0}{dk} = 0 \text{ at } k = k_{c0} \qquad (22)$$

so that the following expression for k_c is obtained

$$k_c = \left[k_0\left(1 + Di\,C\frac{dE}{dk}\right)\right]_{k=k_{c0}} \qquad (23)$$

where $C = -Ra_0/(kd^2Ra_0/dk^2)$. In a similar fashion, the following expression for Ra_c^+ is obtained

$$Ra_c^+ = [Ra_0(1 + Di\,E)]_{k=k_{c0}}. \qquad (24)$$

The evaluation of the terms on the right-hand side of equations (23) and (24) requires the solution of the eigenvalue problem, equations (11a)–(11m). Such solutions were obtained using a Tau scheme [5]. Calculated values of Ra_{c0} and k_{c0} are in very good agreement with those presented in ref. [4]. The integrals in equation (17) were evaluated by a twentieth-order Gaussian quadrature, while the derivatives in equation (23) were evaluated by a second-order central difference scheme.

Table 1. Values of Ra_{c0} and k_{c0} for different values of Bi

Bi	Ra_{c0}	k_{c0}
10^6	1100.649	2.6823
10^1	989.491	2.5889
10^{-3}	669.137	2.0859

Table 2. Values of the different terms appearing in equations (25) and (26) for different values of Bi

Bi	N_1	N_2	$C\frac{dN_1}{dk}$	$C\frac{dN_2}{dk}$
10^6	177.55	−166.19	118.48	−2.44
10^1	156.86	−160.13	104.45	−2.81
10^{-3}	119.34	−128.24	82.39	−4.72

3. DISCUSSION

Before we proceed with the presentation of the results, we first discuss the $Di = 0$ case. Table 1 shows the values of Ra_{c0} and k_{c0} for different values of the Biot number. As the boundary condition, equation (7g), shows, $Bi \to \infty$ and $Bi \to 0$ correspond to fixed temperature and heat flux respectively at the free surface. Table 1 shows that Ra_{c0} decreases monotonically with decreasing Bi. Sparrow *et al.* [4], argue that this is due to the stronger constraint imposed on the temperature perturbation by the fixed temperature condition. Physically speaking, in the constant heat flux case the heat suppled to (rejected by) the perturbation at the rigid wall, does not leave (enter) the free surface. In the constant temperature case, heat is crossing the surface, attenuating thus the carrier disturbance. Table 1 shows that k_{c0} exhibits a similar behavior as Ra_{c0}. Noting that the wavenumber is a measure of the inverse of the distance traveled by a moving particle, this behavior is in agreement with the comments made for the behavior of Ra_{c0}.

Proceeding now with the discussion of the effects of viscous dissipation on the stability of the film flow, equations (23) and (24) are recast for convenience as

$$\frac{k_c - k_{c0}}{k_{c0}} = \Lambda^2 Di\left[\frac{1}{Pr}\left(C\frac{dN_1}{dk}\right) + \left(C\frac{dN_2}{dk}\right)\right]_{k=k_{c0}} \qquad (25)$$

$$\frac{Ra_c - Ra_{c0}}{Ra_{c0}} = \Lambda^2 Di\left[\frac{N_1}{Pr} + N_2\right]_{k=k_{c0}}. \qquad (26)$$

In the above equations, the terms involving N_1 come from the dissipation term in the perturbation energy equation (7e). The numerical results shown in Table 2 indicate that inclusion of this term yields a more stable flow and a higher value for the critical wavenumber. A physical explanation of this behavior can be given as follows. For $\Lambda = 0$ ($\beta = 0$), the gravity component parallel to the plane is zero and as a result a mean flow is not generated. In this case equations

Table 3. Values of Pr_R and Pr_k for different values of Bi

Bi	Pr_R	Pr_k
10^6	1.07	48.55
10^1	0.98	37.17
10^{-3}	0.93	17.45

(7a)–(7i) show that $\hat{u} = 0$. Moreover, with the viscous dissipation term absent from the linear disturbance equations, the stability of the flow depends only on the outcome of competition of the buoyancy and diffusion effects upon the motion of a fluid particle in the y–z plane. For $\Lambda \neq 0$, the motion is three-dimensional ($\hat{u} \neq 0$) so that a fluid particle now travels a larger distance along a spiral in order to complete an orbit. If $Di = 0$, the fact that $\hat{u} \neq 0$ has no effect on the stability of the flow since the instability mechanism described previously is operative only during the vertical motion of the fluid particle. This is shown in equations (7a)–(7i), where for $Di = 0$ the solution to the eigenvalue problem is independent of $\hat{u}$. However, when $Di \neq 0$, part of the energy which is supplied by the buoyancy to the disturbance is not only diffused but is dissipated as well. Therefore, by consuming energy that otherwise would have gone to the kinetic energy of the disturbance, viscous dissipation stabilizes the flow. As equation (26) shows, this stabilization process decreases as the value of the Prandtl number increases. The Prandtl number appears in the governing equations in the x-momentum equation (7b) only and is a measure of the inertia forces in the streamwise direction. As the Prandtl number increases (say by allowing viscosity to increase) these forces diminish and as a result the magnitude of $\hat{u}$ decreases. Thus, the dissipation term in the energy equation (7e) becomes less stabilizing.

The terms involving N_2 come from the convective term in the perturbation energy equation (7e) and represent the effects of the viscous dissipation in modifying the basic temperature gradient. As equation (5b) shows, a non-zero dissipation number yields a steeper temperature gradient in the upper part of the film ($z < 0.63$). Consideration of viscous dissipation in the basic state is equivalent to heat generation across the film. Sparrow *et al.* [4], examined the effects of uniformly distributed internal heat sources in the Rayleigh–Benard problem and showed that uniform internal heating has always a destabilizing influence. The results shown in Table 2 confirm the destabilizing influence of the modified temperature gradient and indicate a decrease of the critical wave-number.

Equations (25) and (26), show that the overall effect of the viscous dissipation on the stability of the flow depends on the Prandtl number. For a given Biot number, there are two values of the Prandtl number, say Pr_R and Pr_k, shown in Table 3, for which viscous dissipation has no effect on Ra_c^+ and k_c, respectively. For $Pr > Pr_R$ the flow will be destabilized, while for $Pr < Pr_R$ the flow will be stabilized. Since the values of Pr_R are very close to unity, we can conclude that viscous dissipation will stabilize liquid metals while it will destabilize all other liquids. This is shown in Table 4 for mercury ($Pr = 0.025$) and water ($Pr = 6$). For $Pr > Pr_k$ the critical wavelength will increase while for $Pr < Pr_k$ it will decrease. The values of Pr_k in Table 3, indicate that the critical wavelength decreases except for the relatively viscous fluids.

Table 4. Values of the sum in square brackets in equation (26) for mercury ($Pr = 0.025$) and water ($Pr = 6$)

Bi	$\frac{N_1}{Pr} + N_2$	
	$Pr = 0.025$	$Pr = 6$
10^6	6935.8	−136.5
10^1	6114.2	−133.9
10^{-3}	4645.3	−108.3

The results in Table 4 show that the effects of viscous dissipation are stronger in the constant temperature case ($Bi \to \infty$) relative to the constant heat flux case ($Bi = 0$). The physical explanation of this behavior follows from the discussion of the effects of Bi for the $Di = 0$ case. As the Biot number decreases, the heat crossing the free surface decreases. As a result, the heat retained by a moving fluid particle increases. Therefore, the heat exchange due to dissipation effects becomes less important.

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EFFETS DE LA DISSIPATION VISQUEUSE SUR LA STABILITE D'UN FILM LIQUIDE DESCENDANT LE LONG D'UN PLAN CHAUD INCLINE

Résumé—On examine les effets de la dissipation visqueuse sur la stabilité linéaire d'un film liquide descendant sur un plan incliné chaud. On montre que pour le mode thermique d'instabilité, la dissipation visqueuse a des influences stabilisantes ou déstabilisantes. L'effet global sur la stabilité de l'écoulement dépend de la valeur du nombre de Prandtl.

EINFLUSS DER VISKOSEN DISSIPATION AUF DIE STABILITÄT EINES FLÜSSIGKEITSFILMES, WELCHER AN EINER BEHEIZTEN, GENEIGTEN PLATTE HERABFLIESST

Zusammenfassung—Der Einfluß der viskosen Dissipation auf die lineare Stabilität eines Flüssigkeitsfilmes, welcher an einer beheizten, geneigten Platte herabströmt, wird analysiert. Dabei zeigt sich, daß die viskose Dissipation für die thermische Seite der Instabilität sowohl stabilisierende wie auch destabilisierende Wirkungen besitzt. Der Einfluß auf die Stabilität der Strömung insgesamt hängt von der Prandtl-Zahl ab.

ВЛИЯНИЕ ВЯЗКОСТНОЙ ДИССИПАЦИИ НА УСТОЙЧИВОСТЬ ПЛЕНКИ ЖИДКОСТИ, СТЕКАЮЩЕЙ ПО НАГРЕТОЙ НАКЛОННОЙ ПЛОСКОСТИ

Аннотация—Исследуется влияние вязкостной диссипации на линейную устойчивость пленки жидкости, стекающей по нагретой наклонной пластине. Показано, что в режиме тепловой неустойчивости вязкостная диссипация оказывает как стабилизирующий, так и дестабилизирующий эффекты. Общая устойчивость течения зависит от величины числа Прандтля.